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WAVE EQUATION ANALYSIS AND DYNAMIC MONITORING

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The advent of the wave equation analysis during the 1960s through the 1970s was a quantum leap in modern foundation engineering. For the first time, the full complex of pile driving could be considered, that is, velocity dependent aspects (damping), soil deformation characteristics, soil resistance (total as well as its distribution of resistance along the pile shaft and between the pile shaft and the pile toe), hammer efficiency, hammer and pile cushions parameters, and other.

However, while the wave equation is theoretically correct, when used without factual data, the results are only qualitatively correct, not quantitatively. Each input datum is really a variable with a certain range of possible values. As there are many input parameters, the combined effect is that the results (for instance, a bearing graph) is only somewhere near the quantitatively correct value. Therefore, one should never just be satisfied with one analysis (computer run), but run several analyses to determine the width of probable conditions through applying ranges of input values. The results are then presented not as one well defined curve, but as representative areas or bands, which widths define the degree of confidence in the validity of the analysis to the actual behavior, as shown in Fig. 1. More than one confidence band could exist, depending on whether, for example, the analysis addresses end-of-initial-driving or restriking conditions.

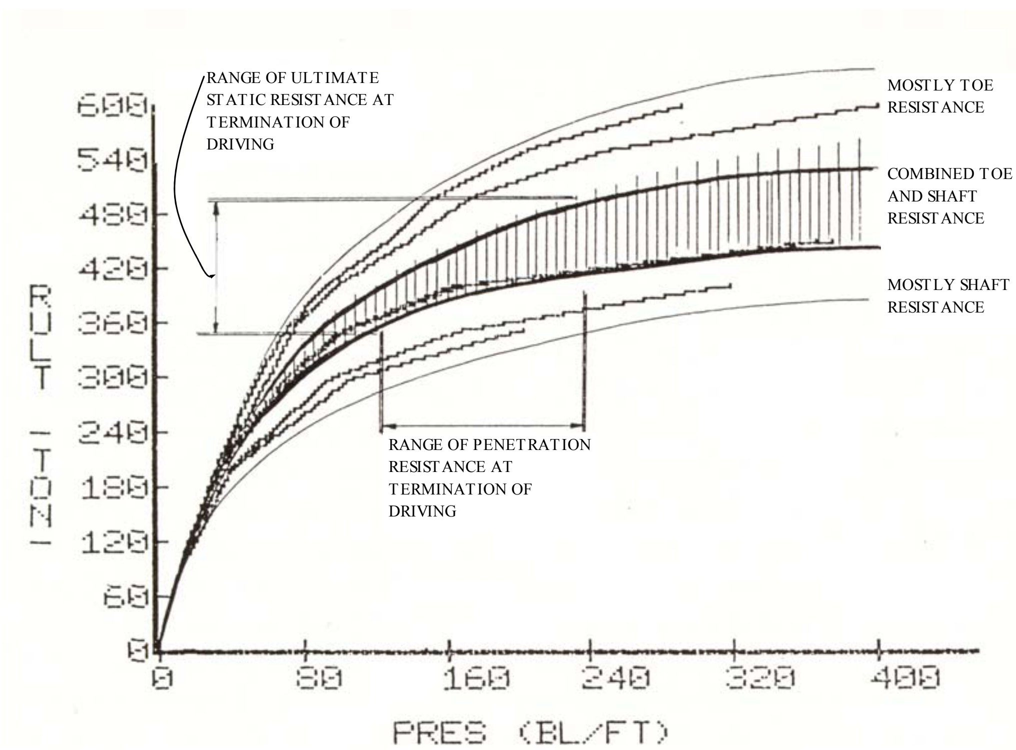

Fig. 1 Bearing Graph from WEAP Analysis

Often, the bearing graph can be combined with empirical information and pared with "engineering judgment" to narrow the width of the confidence band. Usually, though, once one has realized that the wave equation does not give single line or simple answers — in effect realized the necessity of producing a band of curves from the wave equation analysis — it becomes clear that wave equation analysis is a tool that cannot be used alone.

The full power of the wave equation analysis is first realized when combined with dynamic monitoring of the pile during driving. Dynamic monitoring was developed in the USA by Dr. G. G. Goble and co-workers at Case Western University in the early 1970's. It has since been accepted, adopted really, all over the world. The dynamic monitoring consists in principle of attaching gages to the pile shortly below the pile head that measure and record force and velocity induced in the pile by the impact. (There are numerous papers describing the actual system.)

The dynamic measurements are usually presented in the form of "wave traces," which show the measured force and velocity drawn against time, Fig. 2. At first, force and velocity are proportional by the so-called acoustic impedance, a material constant (equal to AE/c, the product of cross sectional area and elastic modulus over the wave propagation velocity). Therefore, when plotted to scale of the ratio of the impedance, force and velocity at first plot on top of one another.

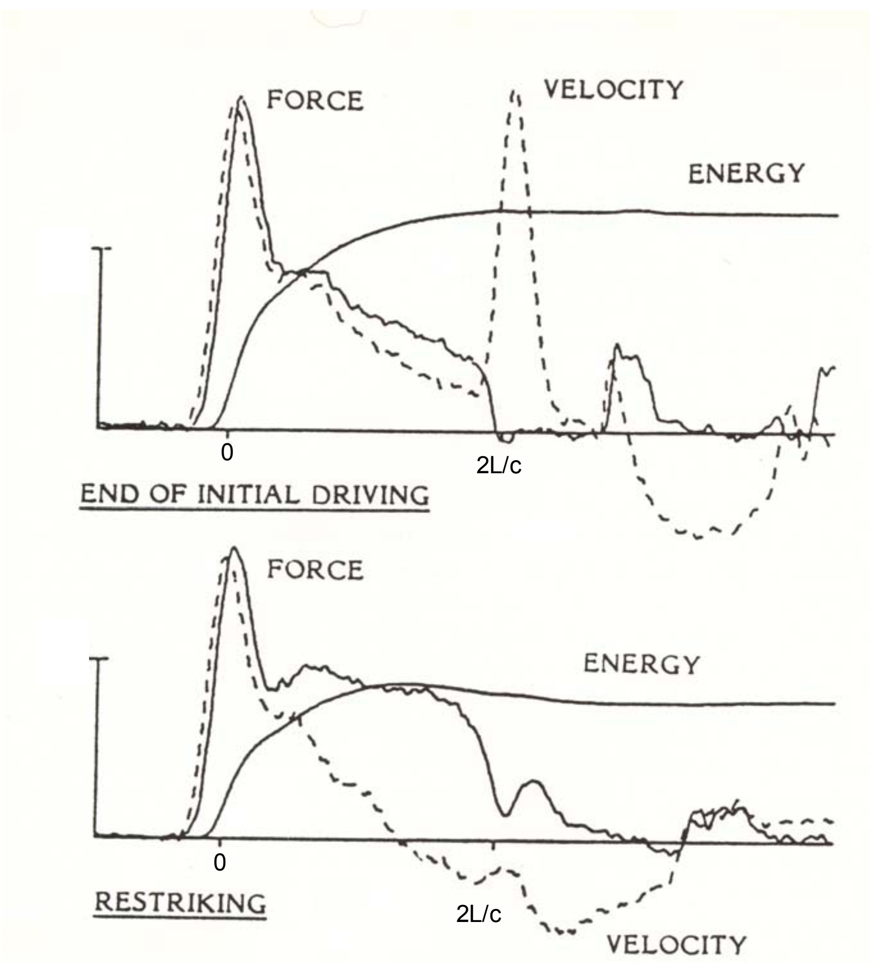

Fig. 2 Example of Wave Traces

Fig. 2 shows traces taken from a pile at the end of initial driving and at restriking. (The force and velocity traces have been supplemented with traces of transferred energy, which is calculated from the force and velocity traces). It should be obvious also to the inexperienced that the pile has been subjected to a capacity increase due to soil set-up although the restriking was performed with a smaller force and energy. The tremendous value of having such records in an actual case is evident.

When the strain wave travels down the pile meeting the soil, soil resistance will cause reflections that travel back up the pile. These will result in an increase of the stress (at the location of the gage) and a simultaneous decrease of velocity (the pile slows down), visible as a separation of the two curves. The greater the separation, the greater the resistance. At the pile toe, if there is no toe resistance, a tensile wave will reflect up the pile, which will be measured as the pile head gages as a reduction of stress and an increase of velocity; that is, the force trace dips and the velocity trace peaks.

The above is illustrated in the upper set of traces given in Fig. 3 showing traces taken from the driving of a 140 feet (43 m) long, 18-inch (457 mm) diameter, octagonal concrete pile. Conventionally, the time scale is given in units of L/c, that is, the time it takes for the strain wave to travel the length of the pile. At 2 L/c, therefore, the traces show the reflections originating from the pile toe. The peak at zero time, zero L/c, is defined as impact. Note that impact force is transmitted to the pile before this time as well as after, i.e., the duration of the transfer is longer than the time it takes for the wave to travel down to the pile toe and be reflected back up to the pile head (the gage location).

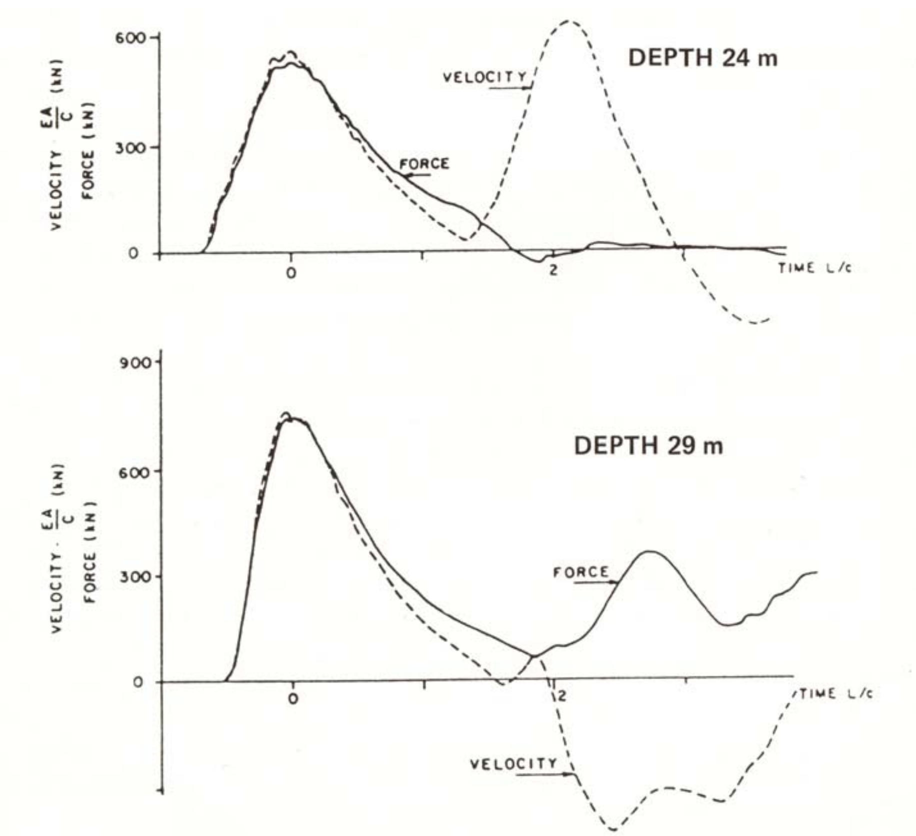

Fig. 3 Wave Traces from Easy driving and from Termination Driving, i.e.,
from driving against little toe resistance and from driving against significant toe resistance

The figure presents two sets of traces**:** traces from easy driving at a depth of 78 feet (24 m) and at approaching termination at 95 feet (29 m). The small separation of the force and velocity traces shown in the upper traces indicate that the pile is affected by only a small amount of shaft resistance and, further, the velocity peak at 2 L/c indicates that there is little or no resistance at the pile toe. The lower set of traces shows, in contrast, a small force peak indicating that the pile toe is now meeting some toe resistance.

The visual study of the wave traces provides very important information to the monitoring engineer. Blow by blow is displayed as the pile is driven. The data are routinely stored on a recorder and can be played back in the office for renewed study.

The data can be treated analytically, of course. The two traces, force and velocity, are mutually independent records. By taking one trace, say the velocity, as an input to a wave equation computer program called CAPWAP, a force-trace can be calculated. The shape of this calculated force trace depends on the actual hammer input given to the pile as represented by the velocity trace and on the static and dynamic soil parameters used as input in the analysis. Because the latter are assumed values, the calculated force-trace may at first attempt be very different from the observed force-trace. However, by adjusting the static and dynamic input data, the calculated force trace can be adjusted to agree better with the measured trace and the match is improved. Ultimately, after a few iteration runs on the computer, the calculated force-trace is made to agree well with the measured trace. A match means that the soil data (quake, damping, and ultimate shaft and toe resistance values) are those of the soil in which the pile has been driven. In other words, the CAPWAP match has calibrated the wave equation analysis. Thereafter, also with changes of pile lengths, hammer unit, and pile size, the wave equation can be used with much greater confidence and the resulting band width can be narrowed considerably.

As an important results of the CAPWAP analysis the static capacity of the pile has been determined. The CAPWAP determined capacity is usually close to the capacity determined in a static loading test. This does not mean that it is identical to the value obtained from a static test. Consider that the capacity of a test pile as evaluated from a load-deformation curve can vary by 20 percent with the definition of failure load applied. Also, only very few static tests can produce load values within a 5 percent error and that the usual inaccuracy lies at about 15 percent or more.

A CAPWAP analysis performed on good measurements taken when a pile penetrates at about 5 blows to 8 blows per inch will provide values of capacity, which are reliable and representative for the static behavior of the pile at the time of the driving. Provided the static test is equally well performed (not always the case) the two values of static capacity are within a 15 percent to 20 percent deviation. For all practical engineering purposes, this can be taken as complete agreement. Building codes in Europe and Canada recognize this and have accepted the dynamic methods as equal to routine static pile testing. As the costs of one conventional static test equal the costs of ten to twenty or more dynamic tests and analyses, the savings can be considerable.

The dynamic monitoring does not replace static testing and conventional inspection methods, it compliments them. Commonly, instead of, say, three static tests, only one is performed and combined with dynamic monitoring and analysis. Thus, money and time is gained along with an improved confidence.

Dynamic monitoring has its share of skeptics. Among the most commonly heard criticism is from persons who have experienced cases when the dynamic measurements, as displayed by the data acquisition unit, the Pile Driving Analyzer, were taken at their mathematical value without consideration to soil information and other influencing factors. However, dynamic monitoring is not a black-box technique replacing the need for knowledge and experience of the persons using the technique.